# Nonlinear time series analysis in production systems

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***Abstract***— We will present a successful application of nonlinear time series analysis to the balancing problem of spinning rotors in machine engineering.
The aim of our exploration is to predict the amount and angular position of the unbalance during the measurement. This can be done with nonlinear forecasting techniques.

## I. Introduction

A rotating body will not exert any variable disturbing force on its supports when the axis of rotation coincides with one of the principal axes of inertia of the body. This condition is quite difficult to achieve in the normal process of manufacturing, because some irregularities in the mass distribution are always present. An unbalanced rotor will cause vibration and stress in the rotating part and in its supporting structure. An asymmetrical distribution of mass leads to vibrations in the axle bearing. This is shown in figure 1. The centrifugal force of the spinning rotor with mass $m$ is given by

$$\vec{F} = \omega^2 (m+u) \cdot \vec{e}_s = \omega^2 u \cdot \vec{r} \ ,$$

In order to remove these vibrations and forces, balancing is necessary. To measure the vibration due to rotor unbalance, a vibration transducer is attached to the axle bearing. The vibration sensor converts the mechanical motion into an electrical signal that corresponds to the rotor's unbalance. The unbalance could be described as a vector in the complex plane [1]. This vector moves over the complex plane as a function of rotor frequency. If a specific number of revolutions is reached, the absolute value of this vector is proportional to the amount of mass imbalance and the phase angle provides information about the location of the mass imbalance. The goal of our exploration is to predict the amount and angular position of the mass imbalance during the measurement.

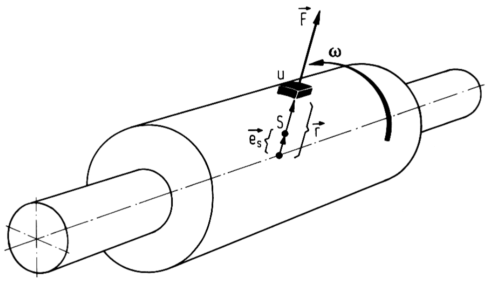


Figure 1: We consider a symmetrical rotor with mass $m$ and an additional mass $u$. The center of gravity $S$ is displaced to $\vec{e}_s$ by a mass $u$ with radius $\vec{r}$, which leads to the centrifugal force $\vec{F}$.

### Modeling the balancing process

The balancing process could be described as complex function of the rotor frequency $\omega$:

$$
\begin{aligned}
U : R &\to C \\
\omega &\mapsto U(\omega) \; .
\end{aligned}
\qquad (1)
$$

The curve $U(\omega)$ in the complex plane is sampled at discrete frequencies $\omega_1, \ldots, \omega_n$. This unbalance measurement can be summarized as a vector $\vec{U}$ in the complex Euclidean space $\mathbf{C}^n$:

$$
\vec{U} = \begin{pmatrix} U(\omega_1) \\ \vdots \\ U(\omega_n) \end{pmatrix} = \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix} . \qquad (2)
$$

An example for a typical measurement is given in figure 2. Now we can formulate our problem: Given the first $k$ values of the measurement $u_1, \ldots, u_k$, we try to predict the last value $u_n$, which represents the imbalance. This prediction could be done by using the *Nearest-Neighbour-Prediction*, which is well known in the field of nonlinear dynamics [2]. In order to predict the time evolution of a given state $\vec{x}_t$ in the phase space of a dynamical system, we choose a neighbourhood of this state and observe the $T$-step time evolution of the states in this neighbourhood. The mean of this evolved states should be a good estimation of $\vec{x}_{t+T}$ (see figure 3). If we want to apply this method to our problem, we have to define a distance between two unbalance measurements. This could be done by using the definition of $\vec{U}$ together with the Euclidean distance in $\mathbf{C}^n$. The distance $d(\vec{U}, \vec{V})$ between two unbalance measurements $\vec{U}$ and $\vec{V} \in \mathbf{C}^n$ is given

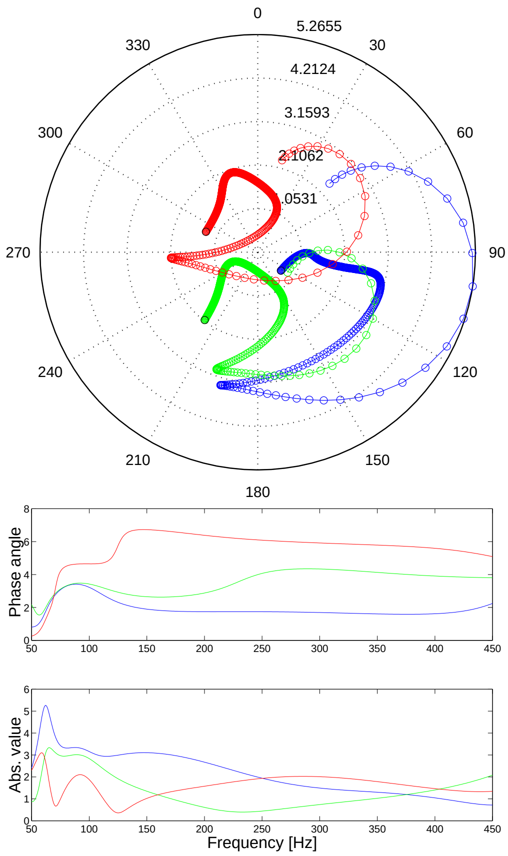


Figure 2: Three typical trajectories of $\vec{U}$ in the complex plane. The lower two diagrams show the absolute value and the phase angle as a function of the rotor frequency.

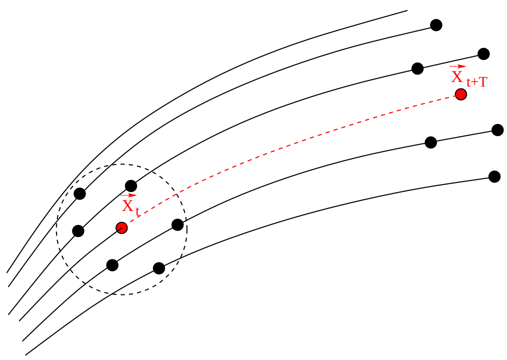

Figure 3: We use the states in the neighbourhood (dotted circle) of $\vec{x}_t$ to predict the time evolution $\vec{x}_{t+T}$.

by:

$$d(\vec{U},\vec{V}) := \|\vec{U}-\vec{V}\|^2 \qquad (3)$$
$$= \sum_{k=1}^{n}(u_k - v_k)\overline{(u_k - v_k)} \, .$$

## Predicting the balancing process

In our case we try to predict the unbalance of wound commutator armatures that are manufactured in large production volumes. Our goal was to shorten the balancing process by using only the first values of the measurement to make a real-time prediction of the expected unbalance. For this purpose we take a database of unbalance measurements and make the assumption, that rotors with similar mass distributions should have similar trajectories in the complex plane. Suppose we want to predict the unbalance of $\vec{U} = (u_1, \dots, u_k)^\dagger$, then we search the nearest-neighbour $\vec{V}^{\mathrm{nn}}$ in our database, referring to the distance $d(\vec{U},\vec{V})$ and we take the unbalance of $\vec{V}^{\mathrm{nn}}$ as an estimate for the unbalance of $\vec{U}$. In figure 4 five examples are shown, illustrating the quality of the prediction.

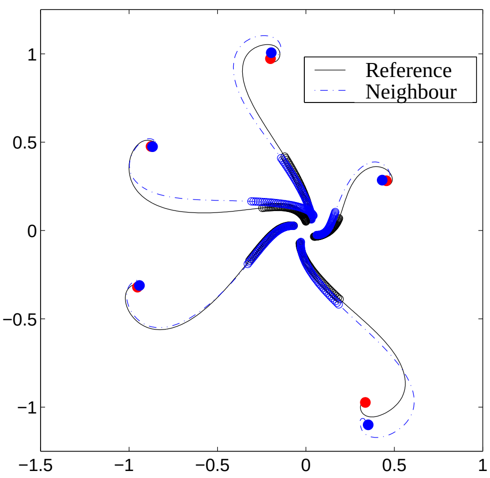


Figure 4: This figure illustrates the prediction method. In order to predict the unbalance of a reference measurement (the dot a the end of the solid line) we use the nearest-neighbour-trajectory (dashed line). Only the first 50 points (plotted as circles) of 200 points are used for the neighbour search.

## Remarks

As we mentioned above, this prediction is used to abbreviate the process of balancing. This could only be done, if the prediction works in real-time. But the problem of neighbour search in high-dimensional spaces is sometimes a time consuming task. So for large databases it is necessary to use fast nearest neighbour algorithms, like the ATRIA-Algorithm (Advanced Triangle Inequality Algorithm) [3].

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## References


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